Adaptive method for Non-Intrusive Spectral Projection: Application on Eddy Current Non Destructive Testing

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Abstract — The Non-Intrusive Spectral Projection (NISP) method is widely used for uncertainty quantification of numerical models. This technique is based on polynomial chaos expansion which requires computation of multidimensional integrals. To evaluate those integrals an automatic adaptive algorithm, based on the nested Gauss-Patterson scheme, has been developed to take into account the importance of each random variable in the model and applied on. Numerical results obtained on an industrial NDT study demonstrate the efficiency of the proposed method with regard to a required accuracy for the first moment's records.

I. INTRODUCTION

For many years, deterministic modelling approach assuming material properties, sources and geometric dimensions to be known, has made it possible to deal with great number of applications in engineering field. However, design, reliability or risk management require more and more to estimate the influence of the uncertainties of the input data on the interest output data [1][2].

One way to take into account the variability of the input data is to consider them as random fields or variables. Among the different approaches, the Non-Intrusive Spectral Projection (NISP) method consists in projecting the stochastic solution in the orthogonal polynomial chaos basis. The chaos expansion coefficients are calculated by evaluating multidimensional integrals on a set of deterministic simulations obtained by sampling methods like full tensor-product quadrature (Gauss, Clenshaw-Curtis) or Smolyak sparse grids. Theses schemes are isotropic formulas in the sense that the different integral directions are discretized in an equal manner. Even with a sparse grid, the number of quadrature points highly increases with the number of input variables. Moreover, integrating in the same way along each stochastic dimension may turn out to be not adapted when the numerical model is sensitive for only one or small number of stochastic dimensions. An adaptive procedure is a way to reduce the number of quadrature points by taking advantage of the difference of sensitivity along the stochastic dimensions [3]. In this work, we propose an adaptive algorithm based on non-isotropic nested Gauss-Patterson formulas taking into account the model global sensitivity to the input random variables. This method has been applied to the Eddy currents Non Destructive Testing inspection of steam generator (SG) tubes with regard to clogging of the quatrefoil support plate (SP) in steam generators of nuclear power plants. This deposit of corrosion products raises a safety concern: to some extent this phenomenon may significantly affect the water and temperature distribution and steam circulation and cause flow induced vibration instability leading to tube cracking risks [6].

II. STOCHASTIC PROBLEM

Let us consider a spatial domain D divided into M1 conducting disjoint subdomains and M2 non conducting disjoint subdomains. Permeability and conductivity of D are assumed to be random fields, written as $\mu(\mathbf{x},\theta)$ and $\sigma(\mathbf{x},\theta)$. $\mathbf{x}$ is a spatial variable and θ denotes the outcome belonging to the random space Θ. Since the behavior laws are random, **B** and **E** are unknown random fields defined on $D\otimes\Theta$, where $\otimes$ denotes the Kronecker product. By using the magnetic vector potential **A** and the electric scalar potential φ formulation, the stochastic magneto-harmonic formulation is given by:

$$\mathbf{curl}\left(\frac{1}{\mu(\mathbf{x},\theta)}\mathbf{curl}\mathbf{A}(\mathbf{x},\theta)\right)+\sigma(\mathbf{x},\theta)\left(j\omega\mathbf{A}(\mathbf{x},\theta)+\mathbf{grad}\,\varphi(x,\theta)\right)=\mathbf{J}_s$$

where ω is the frequency. $\mathbf{A}(\mathbf{x},\theta)$ and $\varphi(\mathbf{x},\theta)$ are defined in $D\otimes\Theta$, To solve the stochastic problem, the non-intrusive spectral projection is used. The method will thereafter be described.

III. NON-INTRUSIVE SPECTRAL PROJECTION

Let us assume that the permeabilities and the conductivities of each subdomain are constant but uniform independent random variables. Considering the Legendre polynomial chaos as the projection basis of the stochastic solutions, the solutions can be expanded as an infinite series:

$$S(x,\theta)=\sum_{\alpha}S_{\alpha}(x)\Psi_{\alpha}(\theta)$$

where $S_\alpha(\mathbf{x})$ are fields functions of space and Ψ_α are multi-variable Legendre polynomials of uniform random variables. Since the set of Ψ_α is an orthogonal basis, the projection coefficients $S_\alpha(\mathbf{x})$ are given by:

$$S_{\alpha}(x)=\frac{E[S(x,\theta)\Psi_{\alpha}(\theta)]}{\left\|\Psi_{\alpha}(\theta)\right\|^{2}}$$

The multidimensional integrals $E[S_\alpha(\mathbf{x},\theta)\Psi_\alpha(\theta)]$ are usually estimated by Gauss formulas, Smolyak sparse grid or sampling methods. In the following, an adaptive and non-isotropic algorithm based on the nested Gauss-Patterson formulas is used instead of computing the projection coefficients.

IV. Adaptive Algorithm

The proposed dimension-adaptive quadrature method is based on the notion of admissible index [4]. At each step of the adaptive process, the method aims at locating the most influent stochastic dimension. The influence of a dimension is estimated with the calculation of local errors which correspond to greatest variation of a criterion combining Sobol indices and the four first moments. Gauss Patterson formulas are used to construct the adaptive nested grid [5]. The general algorithm of the method is described below.

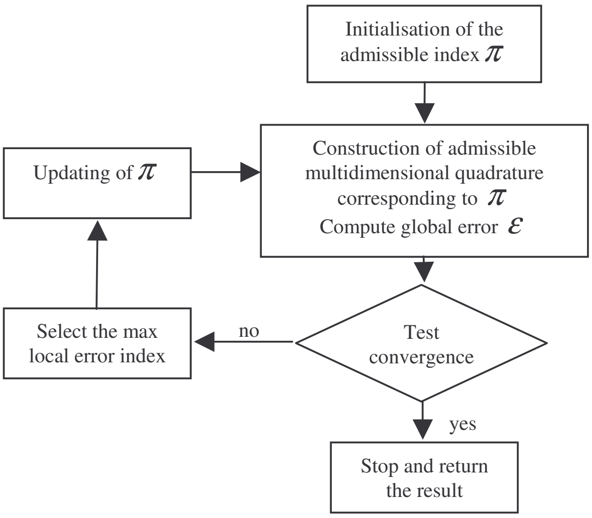

Fig.1 Flow chart of the adaptive procedure.

V. Application

Among the several available techniques to evaluate the amount of clogging in the foils of the support plates (SP) in steam generators, an eddy current inspection technique is under development at EDF [6]. The principle consists in correlating the difference of magnitude of an axial bobbin coil signal at each edge of the Tube Support Plate (TSP) to the amount of deposit in the foils (Fig. 2). As TSP and deposit material properties are not well known, the aim is to evaluate how uncertainties in their permeability and conductivity affect the control signal. According to some experimental data, the relative permeability and conductivity of the TSP (respectively the deposit) have been chosen as independent uniform random variables in the interval [45, 75], (respectively [1.2, 2.8]) and [$1.7*10^6$, $1.8*10^6$], (respectively [60,100] (S/m)). The moments of the differential flux calculated with the adaptive method (see Table I) are compared to the moment calculated with isotropic Gauss-Legendre formulas (see Table II). Results clearly show that the adaptive method makes it possible to get the same accuracy with less quadrature points meaning less computation time.

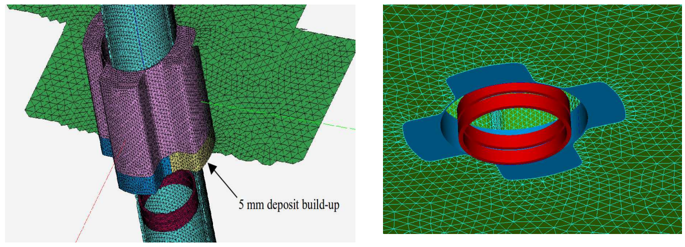

Fig.2 Finite Element model for the clogging of the quatrefoil Support Plate.

TABLE I
EVOLUTION OF DIIFFERENT MOMENT OF THE REAL FLUX PROBABILITY DENSITY OBTAINED BY ISOTROPIC GAUSS-LEGENDRE

Number of simulations	Mean (*e-10 Wb)	Standard deviation (*e-11Wb)	Skewness	Kurtosis
16	3.49	1.52	1.22e-2	1.74
81	3.49	1.05	4.13e-2	1.97
256	3.49	1.05	2.74e-2	1.92

TABLE II
EVOLUTION OF DIIFFERENT MOMENT OF THE REAL FLUX PROBABILITY DENSITY OBTAINED BY THE ADAPTATIVE METHOD

Number of simulations	Mean (*e-10 Wb)	Standard deviation (*e-11Wb)	Skewness	Kurtosis
13	3.39	1.51	3.13e-2	1.93
21	3.49	1.17	1.16e-2	1.89
33	3.49	1.11	2.98e-2	1.91
47	3.49	1.05	2.78e-2	1.91

Conclusion

The proposed adaptive method enables us to find the optimal number of deterministic computations if the order of polynomial chaos is fixed, or to find the optimal order of the chaos if the number of computations is limited. This approach is highly competitive when the model sensitivity to some random variables is negligible.

Acknowledgment

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